Engineering Mass and Force

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1 Intro

Force and mass in SI units have been defined in such a way that it bellies the true complexity invovled when calculating a force on a mass or the resulting mass given a force etc. This becomes immediately apparent to American engineering students and professionals alike when they are given mass in units of ”pounds-mass” instead of slugs. Both of which, by the way, are units of mass—don't let the word pound trick you. But rather than diving right in to what I will call the ”american engineering units” or ”engineering units” for short, I would like to begin with an example using the SI units.
What *is* a Newton? It's defined as the force required to accelerate a kilogram at $1\,\mathrm{m/s^2}$. Since force is defined by $F = ma$, this means one Newton is $1\,\mathrm{kg}\cdot\mathrm{m/s^2}$. Okay, so far so good. But how much does 1 kilogram weigh? Actually, that's an ill-posed question. For what is meant by weight? The mass? The force of gravity on that mass? Which gravity? It is this poorly defined notion of weight that is at the heart of the definition of pounds-force and pounds-mass. From here on, in this text weight will mean the force produced by acceleration due to earth's standard gravity, $9.81\,\mathrm{m/s^2} = 32.2\,\mathrm{ft/s^2}$. Let us suppose we wanted to define a new force called the kilogram-force (kgf). This force we want defined in such a way that, to know the weight of an object, we simply needed to replace the unit kg with kgf. For example, if something is 2.5 kg, then it weighs 2.5 kgf. This can easily be done by defining the kgf as the amount of force required to accelerate an object not by $1\,\mathrm{m/s^2}$ but by the acceleration due to gravity. That is, our kgf is $9.81\,\mathrm{kgm/s^2}$ or 9.81N. If we know an object's weight in Newtons, we can convert to kgf by dividing by 9.81. Notice however that we are *not* dividing by $9.81\,\mathrm{m/s^2}$. 9.81 is simply a unitless conversion factor. All we are doing here is converting from one unit of force to another. The fact our conversion factor shares the same number as the acceleration due to gravity is simply an intended consequence of how we've defined our new unit of force. With this in mind we're ready to tackle the engineering units.

2 Engineering Units

There are in fact two units of force and two units of mass to consider. The units of force are the Poundal and the Pound-Force. The units of mass are the pound-mass and slug. The Poundal is conjugate to the pound-mass, and the pound-force is conjugate to the slug. What this means is that if you use poundals as your unit of force and pounds-mass as your unit of mass, you do not need to do any conversions (likewise for using pounds-force together with slugs). In some sense you can think of pounds-mass and slugs as one system of units, and pounds-force and slugs as a separate system. And just like converting from engineering units system to the SI system requires a conversion factor, using (say) pounds-force with pounds mass also requires a conversion factor because these units are from different systems. This conversion is the subject of section 3. But first, what follows is an introduction to the two systems outlined here.

2.1 Poundals and Pounds-Mass

The pound-mass (lbm) is defined as 0.45359237 kg exactly. From this the poundal (pdl) may be derived as the force that accelerates a pound-mass at a rate of $1\,\mathrm{ft/s^2}$. So we thus have the relation

$$1\,\mathrm{pdl} = 1\,\mathrm{lbm}\cdot\mathrm{ft/s^2}. \tag{1}$$

If you do all your work with poundals and pounds-mass, everything will work out correctly. However, you will invariably wind up having to convert poundals to the more commonly used lbf if you ever want to actually talk to another engineer, so it's ill-advised to do calculations in this units system.
Suppose now that, as in the example with SI units, we wanted to define a unit of force in such a way that given an object's mass in lbm, we could simply replace this mass unit with our new force unit and know the object's weight. That is the pound-force.

2.2 Pounds-Force and the Slug

The pound-force is the force required to accelerate a pound-mass by the acceleration due to gravity, $32.2\,\mathrm{ft/s^2}$. That is,

$$1\,\mathrm{lbf} = 32.2\,\mathrm{lbm}\cdot\mathrm{ft/s^2}. \tag{2}$$

Looking at this relation, we can see that if we define a new unit of mass, the slug, as 32.2 lbm we will have a unit of mass that is accelerated at $1\,\mathrm{ft/s^2}$ by the lbf. This can be seen by using the relation for our new unit of mass

$$1\,\mathrm{slug} = 32.2\,\mathrm{lbm} \tag{3}$$

and substituting this into the relation for lbf, relation (2), to get

$$1\,\mathrm{lbf} = (32.2\,\mathrm{lbm})\cdot\mathrm{ft/s^2} = 1\,\mathrm{slug}\cdot\mathrm{ft/s^2} \tag{4}$$

which tells us a pound-force is the force required to accelerate a slug at a rate of $1\,\mathrm{ft/s^2}$. Thus, if working with pounds-force, it is advisable to use slugs as the unit of mass. As with the poundals/pounds-mass combination, using the combination of pounds-force and slugs allows calculations to be done without any worry of carrying around conversion factors.

3 Unit Coversion and the Pounds-Force—Pounds-Mass system

The problem in engineering however is that often we cannot elect to use a coherent set of units such as pounds-force and slugs or poundals and pounds-mass. Instead, the typical units employed in engineering are the pound-force for force and pound-mass for mass. You might be wondering: what is the usefulness of doing things this way? And rightly so. The utility of this approach is that the value for mass and weight are the same number. Whereas in the SI system to determine the weight of a mass you must multiply by g, with pounds mass this multiplication has effectively been taken care of. The price we pay is that we can no longer use simply $F = ma$, but instead $F = ma/32.2$. This will be discussed in section 3.2.

3.1 Unit Conversions

A useful trick I read in a book once is worth mentioning here, as it's a great way of remembering how to convert units. Let's say we have an object with a mass of 20 lbm. What is this mass in slugs? We can start with relation (3). At the risk of insulting the reader's intelligence, it's crucial to remind here that the equality is saying that what's on the left and right are the same thing. I know that's obvious, but it would be nonsense to claim that $1 = 32.2$. The units to the side of the number are in some sense acting as scalars scaling up 1 to 32.2 or vice versa. They can be thought of as variables storing not a number but something physical. A big rock with a mass of 1 slug would needed to be chopped into 32.2 pieces to have rocks with a mass of a pound-mass.
Returning to our 20 lbm, we have lbm in the numerator. To cancel it out, we can divided both sides of relation (3) by 32.2 lbm to have lbm in the denominator

$$\frac{1\,\mathrm{slug}}{32.2\,\mathrm{lbm}} = 1.$$

Since the right hand side is simply unity, we can multiply anything by it without changing the underlying quantity. And this means we can multiply anything by the left hand side without changing the underlying quantity as well. Thus our 20 lbm becomes

$$20\,\mathrm{lbm} \cdot \left(\frac{1\,\mathrm{slug}}{32.2\,\mathrm{lbm}}\right) = \frac{20}{32.2}\,\mathrm{slugs}$$

3.2 The Pounds-force—Pounds-mass system

Let's step back and recap what we've looked at so far. The pdl/lbm combination is one coherent unit system. The lbf/slug combination is a separate coherent unit system. But the combination of lbf/lbm means that we are working with incoherent units—units from two different systems simultaneously. This means there is very likely a conversion factor that we must include in our equations. We can see this as follows. The equation $F = ma$ is only valid if the unit of force on the left is equivalent to 1 unit of [Mass][Length]/[Time]^2 chosen to be used on the right. Relation (2) tells us that such is not the case if we choose to use lbf as our unit of force and lbm as our unit of mass. We must either convert the pounds-force down to poundals, or the pounds-mass up to slugs. Poundals is never used in practice so we choose to use slugs instead. This means we need to replace m in $F = ma$ with $m * 1\,\mathrm{slug}/32.2\,\mathrm{lbm}$. And this is the reason for using $F = ma/32.2$; it's keeping lbf and converting the lbm into slugs. As a pedagogical aside, if we wanted to stick with lbm we could convert lbf to pdl by using relation (1) to find that

$$32.2\,\mathrm{pdl} = 32.2\,\mathrm{lbm}\cdot\mathrm{ft/s^2} = 1lbf \tag{5}$$

and hence use the equation $32.2F = ma$. But this is never done in practice.
And easy way to remember where the conversion factor goes is to remember that lbm can be replaced by lbf to give an object's weight. From there the mass in slugs can be calculated using the fact that $W = mg$ and thus $m = W/g$ leading to $F = (W/g)a$.