5.32 a) In Example 9, $\vec{B}_{below} = \mu_0 n I \hat{z}$, $\vec{B}_{above} = 0$, and $\vec{K} = nI\,\hat{\phi}$

$\hat{n} = \hat{s}$, so $\vec{K} \times \hat{n} = nI(\hat{\phi} \times \hat{s}) = -nI\hat{z}$

Eq. 76 says $\vec{B}_{above} - \vec{B}_{below} = \mu_0(\vec{K} \times \hat{n})$.

Now, $\vec{B}_{above} - \vec{B}_{below} = 0 - \mu_0 n I \hat{z}$, and

$\mu_0(\vec{K} \times \vec{n}) = \mu_0(-nI\hat{z}) = -\mu_0 n I \hat{z}$, so

Eq. 76 checks for the given configuration of example 9.

B)

In example 11, $A(r,\theta,\phi) = \begin{cases} \frac{\mu_0 R \omega \sigma}{3} r\sin\theta\,\hat{\phi}, & \text{if } r \le R \\ \frac{\mu_0 R^4 \omega\sigma}{3} \frac{\sin\theta}{r^2}\hat{\phi}, & \text{if } r \ge R \end{cases}$

So, $\vec{B} = \vec{\nabla} \times \vec{A} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, A_\phi)\,\hat{r} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi)\,\hat{\theta}$

$$\vec{B}_{above} = \frac{\mu_0 R^4 \omega\sigma}{3}\left[\frac{1}{r^3\sin\theta}\cdot\frac{\partial}{\partial\theta}(\sin^2\theta)\,\hat{r} - \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\sin\theta}{r^2}\right)\hat{\theta}\right]$$

$$\vec{B}_{above} = \frac{\mu_0 R^4\omega\sigma}{3}\left[\frac{2\sin\theta\cos\theta}{r^3\sin\theta}\hat{r} + \frac{1}{r}\frac{1\sin\theta}{r^2}\hat{\theta}\right] = \frac{\mu_0 R^4\omega\sigma}{3r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

$$\vec{B}_{below} = \frac{\mu_0 R\omega\sigma}{3}\left[\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(r\sin^2\theta)\hat{r} - \frac{1}{r}\frac{\partial}{\partial r}(r^2\sin\theta)\,\hat{\theta}\right]$$

$$= \frac{\mu_0 R\omega\sigma}{3}\left[\frac{2r\sin\theta\cos\theta}{r\sin\theta}\hat{r} - \frac{1}{r}2r\sin\theta\,\hat{\theta}\right] = \frac{2\mu_0 R\omega\sigma}{3}\left[\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}\right]$$

$$= \frac{2\mu_0 R\omega\sigma}{3}\hat{z}$$

$\vec{B}$ isn't relevant for this problem, but this is good practice

5.32 Notice $A_{\text{above}}\Big|_{r=R} = \frac{\mu_0 R^4 \omega \sigma}{3} \frac{\sin\theta}{R^2} = \frac{\mu_0 R^2 \omega \sigma}{3} \sin\theta$

$$A_{\text{below}}\Big|_{r=R} = \frac{\mu_0 R \omega \sigma}{3} R\sin\theta = \frac{\mu_0 R^2 \omega \sigma}{3} \sin\theta = A_{\text{above}}\Big|_{r=R}$$

Here, $\hat{n} = \hat{r}$, so

$$\frac{\partial \vec{A}_{\text{above}}}{\partial n} - \frac{\partial \vec{A}_{\text{below}}}{\partial n} = \frac{\partial}{\partial r}\left(\vec{A}_{\text{above}} - \vec{A}_{\text{below}}\right)_{r=R}$$

$$\frac{\partial A_{\text{above}}}{\partial r} = -\frac{2\mu_0 R^4 \omega \sigma}{3} \frac{\sin\theta}{r^3}\hat{\phi} = \frac{-2\mu_0 R \omega \sigma}{3} \sin\theta\hat{\phi}$$

$$\frac{\partial A_{\text{below}}}{\partial r} = \frac{\mu_0 R \omega \sigma}{3} \sin\theta\hat{\phi}$$

$$\frac{\partial \vec{A}_{\text{above}}}{\partial r} - \frac{\partial \vec{A}_{\text{below}}}{\partial r} = -\mu_0 R \omega \sigma \sin\theta\hat{\phi}$$

Since the shell carries uniform surface charge σ,

$\vec{K} = \sigma\vec{v}$. $\vec{v} = (R\sin\theta)\omega\hat{\phi}$, so $K = R\omega\sigma\sin\theta\hat{\phi}$

Hence, per eq. 78,

$$\frac{\partial \vec{A}_{\text{above}}}{\partial n} - \frac{\partial A_{\text{below}}}{\partial n} = -\mu_0\vec{K} = -\mu_0 R\omega\sigma\sin\theta\hat{\phi},$$

so eq. 78 is valid in this instance.

Notice $\vec{K}\times\hat{n} = \vec{K}\times\hat{r} = R\omega\sigma\sin\theta(\hat{\phi}\times\hat{r}) = -R\omega\sigma\sin\theta\hat{\theta}$, and

$$\vec{B}_{\text{above}} - \vec{B}_{\text{below}}\Big|_{r=R} = \mu_0 R\omega\sigma\sin\theta\hat{\theta}$$

So eq. 76 checks as well