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$$\vec{A} = k\hat{\phi} \quad \text{clearly, } \nabla \cdot \vec{A} = 0.$$

So, from eq. (6.2),

$$\vec{J} = -\frac{\nabla^2 \vec{A}}{\mu_0} \quad \text{and by the vector identity,}$$

$$-\nabla^2 \vec{A} = \nabla \times (\nabla \times \vec{A}) - \nabla (\nabla \cdot \vec{A}) = \nabla \times (\nabla \times \vec{A}) \quad \text{since } \nabla \cdot \vec{A} = 0 \text{ in this problem.}$$

$$\nabla \times \vec{A} = \left[ \frac{1}{s} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] \hat{s} + \left[ \frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[ \frac{\partial}{\partial s} [s A_\phi] - \frac{\partial A_s}{\partial \phi} \right] \hat{z}$$

All terms vanish except for  $\frac{1}{s} \left[ \frac{\partial}{\partial s} [s A_\phi] \right] \hat{z} = \frac{1}{s} \left[ \frac{\partial}{\partial s} [s k] \right] \hat{z} = \frac{k}{s} \hat{z}$ .

Then

$$\nabla \times (\nabla \times \vec{A}) = -\frac{\partial}{\partial s} \left[ (\nabla \times \vec{A})_z \right] \hat{\phi} = \frac{k}{s^2} \hat{\phi}$$

$$\text{so } \vec{J} = \frac{\nabla \times (\nabla \times \vec{A})}{\mu_0} = \frac{k}{\mu_0 s^2} \hat{\phi}$$