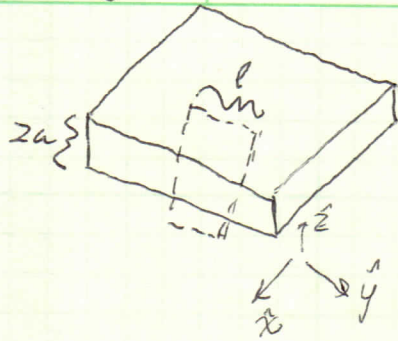


5.15

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \int_S \vec{J} \cdot d\vec{a}$$

First case: $|z| \geq a$

$$\int_S \vec{J} \cdot d\vec{a} = J 2al = 2Jal.$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = B \int_C d\ell = 2Bl$$

$$\left. \begin{array}{l} \int_S \vec{J} \cdot d\vec{a} = J 2al = 2Jal. \\ \oint_C \vec{B} \cdot d\vec{\ell} = B \int_C d\ell = 2Bl \end{array} \right\} \Rightarrow 2Bl = \mu_0 2Jal \Rightarrow B = \mu_0 J a$$

Second case $|z| < a$

$$\int_S \vec{J} \cdot d\vec{a} = -J 2|z|l$$

~~abs b/c area must be positive~~
 no need For abs, $d\vec{a}$ can be negative since it's a vector.

$$\oint_C \vec{B} \cdot d\vec{\ell} = B \int_C d\ell = 2Bl \Rightarrow 2Bl = -2\mu_0 J l |z| \Rightarrow \vec{B} = -\mu_0 J |z| \hat{y}$$

$$\text{so } \vec{B} = \begin{cases} -\mu_0 J z \hat{y} & \text{if } z < a \\ -\mu_0 J a \hat{y} & \text{if } z > a \\ +\mu_0 J a \hat{y} & \text{if } z < -a \end{cases}$$