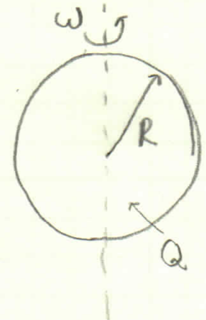
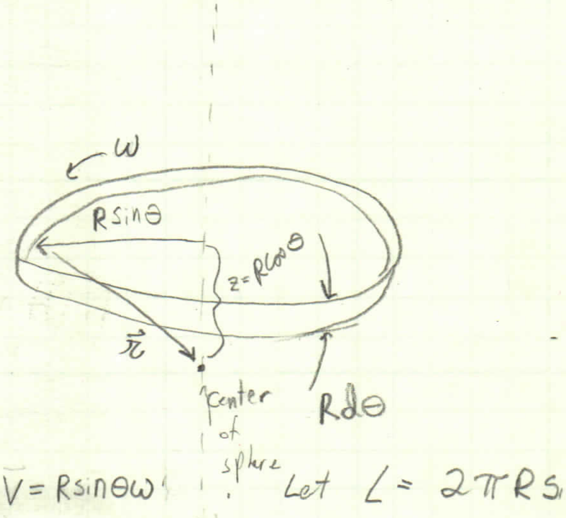
Kimble, Zachary | EM Griffiths | 07/30/18 | 1/1

5.12



$$\sigma = \frac{-Q}{4\pi R^2}$$



$$B(z) = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}$$

$V = R\sin\theta\,\omega$ Let $L = 2\pi R\sin\theta$

$\vec{I} = v\lambda = v\frac{q}{L}$ ← length of line charge

$$d\vec{B} = \frac{\mu_0\,dI}{2}\frac{R^2\sin^2\theta}{(R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}}\hat{z}$$

$dq = 2\pi R^2\sigma\sin\theta\,d\theta = \frac{Q\sin\theta\,d\theta}{2}$

$dI = v\frac{dq}{L} = R\sin\theta\,\omega\,\frac{2\pi R^2\sin\theta\,\sigma\,d\theta}{2\pi R\sin\theta}$

$$d\vec{B} = \frac{\mu_0\,dI}{2}\frac{\sin^2\theta}{R}\hat{z}$$

$dI = R\omega\sin\theta \cdot R\sigma\,d\theta = R^2\omega\sin\theta\,\sigma\,d\theta$

$dI = \frac{Q\omega\sin\theta\,d\theta}{4\pi}$

$$d\vec{B} = \frac{\mu_0}{2R}\frac{Q\omega\sin^3\theta\,d\theta}{4\pi}\hat{z} \Rightarrow \vec{B} = \frac{\mu_0 Q\omega}{8\pi R}\int_0^\pi \sin^3\theta\,d\theta\,\hat{z}$$

$$\int_0^\pi \sin^3\theta\,d\theta = \int_0^\pi\left(\frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta\right)d\theta = -\left[\frac{3}{4}\cos\theta - \frac{1}{12}\cos 3\theta\right]_0^\pi = -\left[\frac{-3}{4} + \frac{1}{12} - \left(\frac{3}{4} - \frac{1}{12}\right)\right]$$

$$= -\left[-\frac{3}{2} + \frac{1}{6}\right] = \frac{9}{6} - \frac{1}{6} = \frac{8}{6} = \frac{4}{3}$$

$$\vec{B} = \frac{\mu_0 Q\omega}{8\pi R}\left(\frac{4}{3}\right)\hat{z} = \frac{\mu_0 Q\omega}{6\pi R}\hat{z}$$