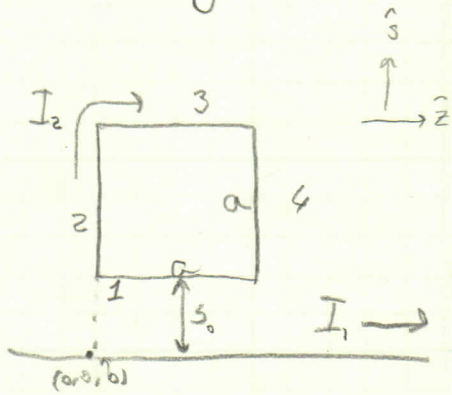
5.10 a)



$$\vec{F} = I_2 \int d\vec{l}' \times \vec{B}$$

From Example 5, $\vec{B} = \frac{\mu_0 I_1}{2\pi s}\hat{\phi}$

$$\left.\begin{aligned} d\vec{l}_{2,4} &= ds\,\hat{s} \\ d\vec{l}_{1,3} &= dz\,\hat{z} \end{aligned}\right\} \Rightarrow d\vec{l}_{2,4} \times \vec{B} = \frac{\mu_0 I_1}{2\pi}\frac{ds}{s}(\hat{s}\times\hat{\phi}) = \frac{\mu_0 I_1}{2\pi}\frac{ds}{s}\hat{z}$$

$$d\vec{l}_{1,3} \times \vec{B} = \frac{\mu_0 I_1}{2\pi s}dz(\hat{z}\times\hat{\phi}) = -\frac{\mu_0 I_1}{2\pi s}dz\,\hat{s}$$

$$I_2\int d\vec{l}_1' \times \vec{B} = -\frac{\mu_0 I_1 I_2}{2\pi s_0}\hat{s}\int_{z=a}^{z=0} dz = -\frac{\mu_0 I_1 I_2}{2\pi}\left(\frac{a}{s_0}\right)\hat{s}$$

$$I_2\int d\vec{l}_3' \times \vec{B} = -\frac{\mu_0 I_1 I_2}{2\pi(s_0+a)}\hat{s}\int_{z=0}^{z=a} dz = -\frac{\mu_0 I_1 I_2}{2\pi}\left(\frac{a}{s_0+a}\right)\hat{s}$$

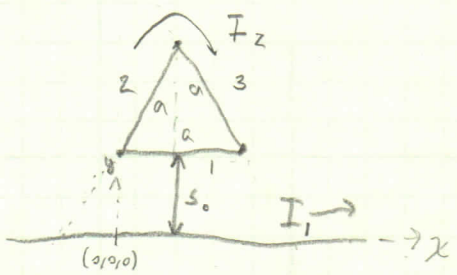
$$I_2\int d\vec{l}_2' \times \vec{B} = \frac{\mu_0 I_1 I_2}{2\pi}\hat{z}\int_{s=s_0}^{s=s_0+a}\frac{ds}{s} = \frac{\mu_0 I_1 I_2}{2\pi}\ln\left(\frac{s_0+a}{s_0}\right)$$

$$I_2\int d\vec{l}_4 \times \vec{B} = \frac{\mu_0 I_1 I_2}{2\pi}\hat{z}\int_{s=s_0+a}^{s=s_0}\frac{ds}{s} = \frac{\mu_0 I_1 I_2}{2\pi}\ln\left(\frac{s_0}{s_0+a}\right) = -\frac{\mu_0 I_1 I_2}{2\pi}\ln\left(\frac{s_0+a}{s_0}\right)$$

with $I_1 = I_2 = I$,

$$F = \frac{\mu_0 I^2}{2\pi}\left[\left(\frac{a}{s_0} - \frac{a}{s_0+a}\right) = \frac{aI^2\mu_0}{2\pi}\left(\frac{1}{s_0} - \frac{1}{s_0+a}\right)\left(\frac{s_0}{s_0}\right)\right]\hat{z}$$

5.10 b)



$$F = I_2 \int d\vec{l}' \times \vec{B}$$

$$\vec{B} = \frac{\mu_0 I_1}{2\pi y}\hat{z}$$

$$d\vec{l}_1' = dx\,\hat{x}$$

$$d\vec{l}_2 = dx\,\hat{x} + dy\,\hat{y}$$

$$d\vec{l}_3' = d\vec{l}_2'$$

for $d\vec{l}_2'$, $y = \sqrt{3}x + s_0 \Rightarrow dy = \sqrt{3}\,dx \Rightarrow \frac{dx}{y} = \frac{dy}{\sqrt{3}y}$

for $d\vec{l}_3$, $y = -\sqrt{3}x + s_0 + 2a \Rightarrow dy = -\sqrt{3}\,dx \Rightarrow \frac{dx}{y} = -\frac{dy}{\sqrt{3}y}$

$$d\vec{l}_1' \times \vec{B} = \frac{\mu_0 I_1}{2\pi y} dx(\hat{x}\times\hat{z}) = -\frac{\mu_0 I}{2\pi y} dx\,\hat{y}$$

$$d\vec{l}_2' \times \vec{B} = \frac{\mu_0 I_1}{2\pi}\left[\frac{dx}{y}(\hat{x}\times\hat{z}) + \frac{dy}{y}(\hat{y}\times\hat{z})\right] = \frac{\mu_0 I_1}{2\pi}\left[\frac{dy}{y}\hat{x} - \frac{dx}{y}\hat{y}\right]$$

$$d\vec{l}_3' \times \vec{B} = d\vec{l}_2' \times \vec{B}$$

$$F_1 = I_2 \int d\vec{l}_1' \times \vec{B} = -\frac{\mu_0 I_1 I_2}{2\pi y}\int_{x=a}^{x=0} dx\,\hat{y}\,\Big|_{y=s_0} = \frac{\mu_0 I_1 I_2}{2\pi s_0} a\,\hat{y}$$

$$F_2 = I_2 \int d\vec{l}_2' \times \vec{B} = -\frac{\mu_0 I_1 I_2}{2\pi}\int_{s_0}^{s_0+\frac{\sqrt{3}}{2}a} \frac{dy}{y}\hat{x} - \frac{\mu_0 I_1 I_2}{2\pi}\int_{s_0}^{s_0+\frac{\sqrt{3}}{2}a} \frac{dy}{\sqrt{3}y}\hat{y} = \frac{\mu_0 I_1 I_2}{2\pi}\left[\hat{x}\ln\left(\frac{s_0+\frac{\sqrt{3}}{2}a}{s_0}\right) - \frac{\hat{y}}{\sqrt{3}}\ln\left(\frac{s_0+\frac{\sqrt{3}}{2}a}{s_0}\right)\right]$$

$$= \frac{\mu_0 I_1 I_2}{2\pi}\left[\ln\left(\frac{s_0+\frac{\sqrt{3}}{2}a}{s_0}\right)\hat{x} - \frac{1}{2}\ln\left(\frac{s_0+\frac{\sqrt{3}}{2}a}{s_0}\right)\hat{y}\right]$$

$$F_3 = I_2 \int d\vec{l}_3' \times \vec{B} = \frac{\mu_0 I_1 I_2}{2\pi}\int_{s_0+\frac{\sqrt{3}}{2}a}^{s_1} \frac{dy}{y}\hat{x} + \int_{s_0+\frac{\sqrt{3}}{2}a}^{s_0} \frac{dy}{2y}\hat{y} = \frac{\mu_0 I_1 I_2}{2\pi}\left[-\int_{s_0}^{s_0+\frac{\sqrt{3}}{2}a}\frac{dy}{2y}\hat{y} + \int_{s_0}^{s_0+\frac{\sqrt{3}}{2}a}\frac{dy}{y}\hat{x}\right]$$

$$\therefore F_2 + F_3 = -\frac{\mu_0 I_1 I_2}{\pi}\int_{s_0}^{s_0+\frac{\sqrt{3}}{2}a}\frac{dy}{2y}\hat{y} = -\hat{y}\frac{\mu_0 I_1 I_2}{\pi\sqrt{3}}\ln\left(\frac{s_0+\frac{\sqrt{3}}{2}a}{s_0}\right) = -\hat{y}\frac{\mu_0 I_1 I_2}{\pi\sqrt{3}}\ln\left(1+\frac{\sqrt{3}a}{2s_0}\right)$$

s_0, with $I_1 = I_2 = I$

$$\vec{F} = \frac{\mu_0 I^2}{2\pi}\left[\frac{a}{s_0}\hat{x} - \frac{\hat{y}}{\sqrt{3}}\ln\left(1+\frac{\sqrt{3}a}{2s_0}\right)\right]$$