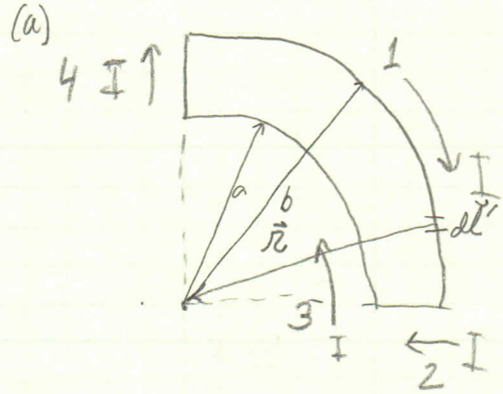
(a)



$$F = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}' \times \hat{r}}{r^2}$$

$$d\vec{l}'_1 = -b\,d\phi\,\hat{\phi} \qquad d\vec{l}'_2 = ds\,\hat{s}$$

$$d\vec{l}'_3 = a\,d\phi\,\hat{\phi} \qquad d\vec{l}'_4 = ds\,\hat{s}$$

$$\vec{r} = -s\hat{s} \Rightarrow r^2 = s^2,\ \hat{r} = \hat{s}$$

clearly $d\vec{l}'_2 \times \hat{r} = d\vec{l}'_4 \times \hat{r} = 0$

$$\frac{d\vec{l}'_1 \times \hat{r}}{r^2} = \frac{b\,d\phi\,(\hat{\phi} \times \hat{s})}{s^2} = \frac{b\,d\phi}{s^2}\hat{z}$$

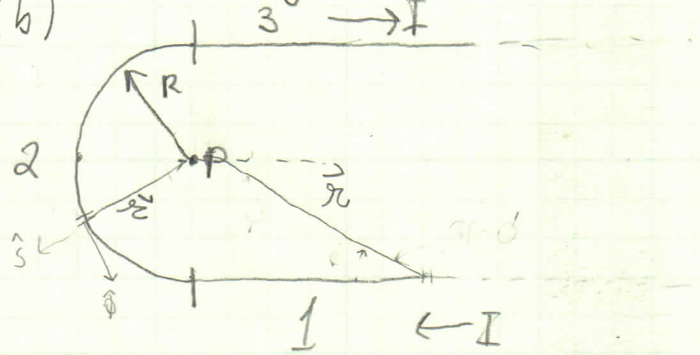
$$\frac{d\vec{l}'_3 \times \hat{r}}{r^2} = \frac{a\,d\phi\,(\hat{\phi} \times \hat{s})}{s^2} = \frac{a\,d\phi}{s^2}\hat{z}$$

$$F = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}'_1 \times \hat{r}}{r^2} + \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}'_3 \times \hat{r}}{r^2}$$

$$\frac{4\pi F}{\mu_0 I} = \int_{\frac{\pi}{2}}^{0} \frac{b\,d\phi}{s^2}\hat{z}\Big|_{s=b} + \int_{0}^{\frac{\pi}{2}} \frac{a\,d\phi}{s^2}\hat{z} = \left[-\frac{\pi}{2b} + \frac{\pi}{2a}\right]\hat{z} = \frac{\pi}{2}\hat{z}\left[\frac{b-a}{ab}\right]$$

$$\vec{F} = \frac{\mu_0 I}{8ab}(b-a)\hat{z}$$

5.9 (b)



$$F = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$d\vec{l}'_1 = d\vec{l}'_3 = dx\,\hat{x}$$

$$d\vec{l}'_2 = R\,d\phi\,\hat{\phi}$$

$$\vec{\mathscr{r}}_{1,3} = \pm R\hat{y} - x\hat{x} \quad \mathscr{r}^2_{1,3} = R^2 + x^2 \quad ; \quad \hat{\mathscr{r}}_{1,3} = \frac{\pm R\hat{y} + x\hat{x}}{\sqrt{R^2+x^2}}$$

$$d\vec{l}'_{1,3} \times \hat{\mathscr{r}}_{1,3} = \frac{\pm R\,dx(\hat{x} \times \hat{y})}{\sqrt{R^2+x^2}} = \frac{\pm R\,dx}{\sqrt{R^2+x^2}}\hat{z}$$

$$\frac{4\pi F_1}{\mu_0 I} = \int_{\infty}^{0} \frac{R\,dx}{(R^2+x^2)^{3/2}}\hat{z} \quad \text{let } x = R\tan\theta \Rightarrow dx = R\sec^2\theta\, d\theta$$

$$\theta_1 = \arctan\left(\frac{x_1}{R}\right) = \frac{\pi}{2} \quad \theta_2 = \arctan\left(\frac{x_2}{R}\right) = 0$$

$$\frac{4\pi F_1}{\mu_0 I} = -\int_0^{\frac{\pi}{2}} \frac{R^2\sec^2\theta\, d\theta}{R^3(1+\tan^2\theta)^{3/2}}\hat{z} = \frac{-1}{R}\int_0^{\frac{\pi}{2}} \frac{\sec^2\theta\, d\theta}{(\sec^2\theta)^{3/2}}\hat{z} = \frac{-1}{R}\int_0^{\frac{\pi}{2}} \cos\theta\, d\theta\,\hat{z} = -\frac{\sin\theta}{R}\hat{z}\Big|_0^{\frac{\pi}{2}} = \frac{-1}{R}\hat{z}$$

$$\frac{4\pi F_3}{\mu_0 I} = -\int_0^{\infty} \frac{R\,dx}{(R^2+x^2)^{3/2}}\hat{z} = \frac{-1}{R}\int_0^{\frac{\pi}{2}} \cos\theta\, d\theta\,\hat{z} = -\frac{\sin\theta}{R}\hat{z}\Big|_0^{\frac{\pi}{2}} = -\frac{\hat{z}}{R}$$

$$\vec{\mathscr{r}}_2 = R\hat{s} \Rightarrow \mathscr{r}^2 = R^2 \quad \hat{\mathscr{r}} = \hat{s}$$

$$d\vec{l}'_2 \times \hat{\mathscr{r}} = R\,d\phi\,(\hat{\phi} \times \hat{s}) = -R\,d\phi\,\hat{z} \qquad \frac{d\vec{l}'_2 \times \hat{\mathscr{r}}}{\mathscr{r}^2} = -\frac{R\,d\phi\,\hat{z}}{R} = -\frac{d\phi}{R}\hat{z}$$

$$\frac{4\pi F_2}{\mu_0 I} = -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\vec{l}'_2 \times \hat{\mathscr{r}}}{\mathscr{r}^2} = \frac{-1}{R}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi\,\hat{z} = -\frac{\phi}{R}\hat{z}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = -\frac{\frac{\pi}{2}+\frac{\pi}{2}}{R}\hat{z} = -\frac{\pi}{R}\hat{z}$$

$$F = \frac{-\mu_0 I}{4\pi}\left[\frac{2}{R}\hat{z} + \frac{\pi}{R}\hat{z}\right] = -\frac{\mu_0}{4\pi R}I(2+\pi)\hat{z} = \frac{-\mu_0}{4R}I\left(1+\frac{2}{\pi}\right)\hat{z}$$