5.6

a) σ = constant. $\vec{K} = \sigma\vec{v}$. $\vec{v}(r) = r\omega\hat{\phi}$ (here I have used the convention that CCW rotations are positive).

so $K = \sigma r\omega\hat{\phi}$

b) $\rho = \dfrac{3Q}{4\pi R^3}$

$\vec{J} = \rho\vec{v}(r,\theta,\phi)$; $\vec{v}(r,\theta,\phi) = r\sin\theta\,\omega\hat{\phi}$

so $\vec{J} = \dfrac{3Q\,r\sin\theta}{4\pi R^3}\omega\hat{\phi}$

5.7

NTS $\int_V \vec{J}\,d\tau = d\vec{p}/dt$ where $\vec{p}$ is the total dipole moment

Consider $\int_V \vec{\nabla}\cdot(x\vec{J})\,d\tau$

$\vec{\nabla}\cdot(x\vec{J}) = J_x + x\dfrac{\partial J_x}{\partial x} + x\dfrac{\partial J_y}{\partial y} + x\dfrac{\partial J_z}{\partial z} = J_x + x(\vec{\nabla}\cdot\vec{J})$

$= J_x - x\left(\dfrac{\partial\rho}{\partial t}\right)$

but $\int_V \vec{\nabla}\cdot(x\vec{J})\,d\tau = \oint_S x\vec{J}\cdot d\vec{a}$ where S is the surface of V

but we know that V is large enough such that all charges and currents are inside, so it is clear that $\oint_S x\vec{J}\cdot d\vec{a} = 0$ since $\vec{J}\big|_{\text{surface}} = 0$.

so $\int_V \vec{\nabla}\cdot(x\vec{J})\,d\tau = \int_V J_x\,d\tau - \int_V x\dfrac{\partial\rho}{\partial t} = 0$

and also, clearly, $\int_V J_y\,d\tau - \int_V y\dfrac{\partial\rho}{\partial t} = 0$

$\int_V J_z\,d\tau - \int_V z\dfrac{\partial\rho}{\partial t} = 0$

i.e. $\hat{x}\int_V J_x\,d\tau + \hat{y}\int_V J_y\,d\tau + \hat{z}\int_V J_z\,d\tau = \int_V \vec{J}\,d\tau = \hat{x}\int_V x\dfrac{\partial\rho}{\partial t} \ldots = \int_V \vec{r}\dfrac{\partial\rho}{\partial t} = \dfrac{d}{dt}\int_V \vec{r}\rho$

and $\vec{P} = \int \vec{r}\rho$ so $\int_V \vec{J}\,d\tau = \dfrac{d\vec{P}}{dt}$ if V encloses all charges and currents