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5.5 a) $K = \sigma \vec{v}$; $\vec{I} = \lambda \vec{v}$. Without loss of generality, suppose the wire has length L and charge Q. Then $\lambda = \frac{Q}{L}$

$\sigma = \frac{Q}{2\pi a L}$, and $\vec{v} = \frac{\vec{I}}{\lambda} = \frac{\vec{I} L}{Q}$ so

$K = \frac{Q}{2\pi a L} \cdot \frac{\vec{I} L}{Q} = \frac{\vec{I}}{2\pi a}$. Alternatively, $K = \frac{d\vec{I}}{d\ell_\perp} \Rightarrow \int d\vec{\ell}\, \overset{\text{constant}}{K} = \int d\vec{I} = \vec{I}$

$d\vec{\ell}_\perp = a\,d\theta \Rightarrow K\int_0^{2\pi} a\,d\theta = 2\pi a K \Rightarrow 2\pi a K = \vec{I} \Rightarrow K = \frac{\vec{I}}{2\pi a}$

b) $J \sim \frac{1}{s}$; introduce constant of proportionality, k, to acheive equality:

$J(s) = \frac{k}{s}$. Now $J = \frac{d\vec{I}}{da_\perp} \Rightarrow \int J\,da_\perp = \int d\vec{I} = \vec{I}$

$\int J\,da_\perp = \int_0^{2\pi}\int_0^a \frac{k}{s} s\,ds\,d\theta = 2\pi k a \Rightarrow 2\pi k a = \vec{I} \Rightarrow k = \frac{\vec{I}}{2\pi a}$

So $J(s) = \frac{\vec{I}}{2\pi a s}$