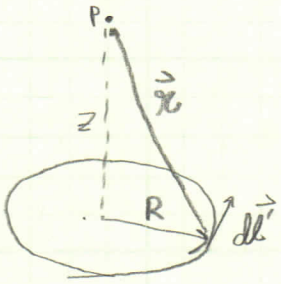
5 Ex 6



$$\vec{r} = -R\hat{s} + z\hat{z} \qquad \vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}' \times \hat{r}}{r^2}$$

$$r^2 = R^2 + z^2$$

$$\hat{r} = \frac{-R\hat{s} + z\hat{z}}{\sqrt{R^2+z^2}} \qquad d\vec{l}' = R\,d\phi\,\hat{\phi}$$

$$d\vec{l}' \times \hat{r} = \frac{-R^2(\hat{\phi}\times\hat{s}) + Rz(\hat{\phi}\times\hat{z})}{\sqrt{R^2+z^2}}\,d\phi$$

$$= \frac{R^2\hat{z} + Rz\hat{s}}{\sqrt{R^2+z^2}}$$

So

$$\vec{B}(z\hat{z}) = \frac{\mu_0 I}{4\pi}\int \frac{R^2\hat{z} + Rz\hat{s}}{(R^2+z^2)^{3/2}}\,d\phi = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}\hat{z}$$