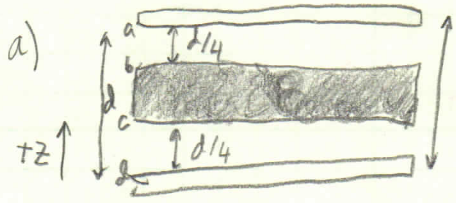
Kinkle, Zachary | EM Griffiths | 10/18/18 |

4.19

a)



$V \rightarrow$ Voltage of plates w/o dielectric

Since the space b/w surfaces a & b, and c&d are vacuum, their voltage sums to $\frac{V}{2}$.

In the dielectric however $\frac{V}{2}$ is reduced by a factor of $\frac{1}{\varepsilon_r}$ so

$$V_1 = \frac{V}{2}\left(1 + \frac{1}{\varepsilon_r}\right) = \frac{V}{2}\left(\frac{\varepsilon_r + 1}{\varepsilon_r}\right) = V\left(\frac{\varepsilon_r+1}{2\varepsilon_r}\right) \text{ and so}$$

$$C = \frac{Q}{V} \quad , \quad C_1 = \frac{Q}{V_1} \Rightarrow \frac{C_1}{C} = \frac{V}{V_1} = V\left(\frac{2\varepsilon_r}{\varepsilon_r+1}\right)\frac{1}{V} = \frac{2\varepsilon_r}{\varepsilon_r+1}$$

in the vacuum region $\vec{D} = \varepsilon_0 \vec{E}$ and $\vec{E}_{ab} = -\nabla V_{ab} = -\left([d - d/4] - d\right)^{-1}\frac{V}{4}\hat{z} = -\frac{V}{d}\hat{z}$

and $V = \frac{2\varepsilon_r V_1}{1+\varepsilon_r}$ so $E_{ab} = -\frac{2V_1}{d}\left(\frac{\varepsilon_r}{1+\varepsilon_r}\right)\hat{z}$

likewise, $E_{cd} = -\frac{2V_1}{d}\left(\frac{\varepsilon_r}{1+\varepsilon_r}\right)\hat{z}$ and hence

$\vec{D}_{ab} = \vec{D}_{cd} = -\frac{2V_1}{d}\left(\frac{\varepsilon_0\varepsilon_r}{1+\varepsilon_r}\right)\hat{z}$. Inside the dielectric,

$$\vec{E}_{bc} = -\nabla V_{bc} = -\frac{V}{2\varepsilon_r}\left([d/4 - [d - d/4]]\right)^{-1}\hat{z} = -\frac{V}{2\varepsilon_r}\left(-d/2\right)^{-1}\hat{z} = -\frac{V}{d\varepsilon_r}\hat{z}$$

$$= -\frac{2V_1}{d(1+\varepsilon_r)}\hat{z} \quad ; \quad \vec{D}_{bc} = \varepsilon\vec{E}_{bc} = \varepsilon_0\varepsilon_r\vec{E}_{bc} = \frac{2\varepsilon_0\varepsilon_r V_1}{d(1+\varepsilon_r)}\hat{z}$$

Using $\vec{P} = \varepsilon_0 \chi_e \vec{E} = \varepsilon_0(\varepsilon_r - 1)\vec{E}$

$$\vec{P}_{ab} = \vec{P}_{cd} = \vec{0}$$

$$\vec{P}_{bc} = \left(\frac{1-\varepsilon_r}{1+\varepsilon_r}\right)\frac{2\varepsilon_0 V_1}{d}\hat{z}$$

4.19 On the plates, taking a gaussian box, w/ relevant area A,

$$EA = \frac{\sigma A}{\varepsilon_0} \Rightarrow \varepsilon_0 E = \sigma \Rightarrow -\sigma_b = \sigma_a = \varepsilon_0 E_{ab} = \frac{2V_1}{d}\left(\frac{\varepsilon_0 \varepsilon_r}{1+\varepsilon_r}\right)$$

on the dielectric, on the top surface,

$$\sigma_b A = -AD_{bc} \Rightarrow \sigma_b = D_{bc} = -\frac{2V_1}{d}\left(\frac{\varepsilon_0 \varepsilon_r}{1+\varepsilon_r}\right)$$

and on the bottom surface

$$\sigma_c A = AD_{bc} \Rightarrow \sigma_c = D_{bc} = \frac{2V_1}{d}\left(\frac{\varepsilon_0 \varepsilon_r}{1+\varepsilon_r}\right)$$

Meanwhile, the bound charges are given by $\vec{P}\cdot\hat{n}$ and so

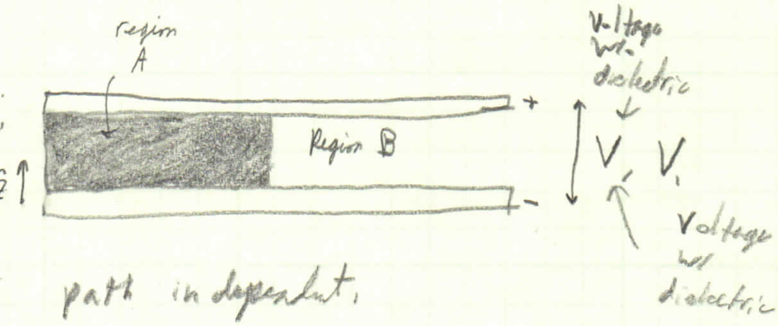
$$\rho_a = \vec{P}_{ab}\cdot\hat{n} = 0$$

$$\sigma_b = \vec{P}_{bc}\cdot\hat{n} = \left(\frac{1-\varepsilon_r}{1+\varepsilon_r}\right)\frac{2\varepsilon_0 V_1}{d}$$

$$\sigma_c = \vec{P}_{bc}\cdot\hat{n} = \left(\frac{\varepsilon_r - 1}{1+\varepsilon_r}\right)\frac{2\varepsilon_0 V_1}{d}$$

$$\sigma_d = \vec{P}_{cd}\cdot\hat{n} = 0$$

Next, for this configuration:



We know that Votage is path independent, so the Votage across region B is the same as across region A, namely, V_1. We know the dieletric's Voltage is reduced by ε_r so

$$V_1 = \frac{V}{\varepsilon_r} \Rightarrow C = \frac{Q}{V}, \quad C_1 = \frac{Q_1}{V_1} \Rightarrow \frac{C_1}{C} = \frac{Q_1}{Q}\frac{V}{V_1} = \varepsilon_r \frac{Q_1}{Q}$$

Since the voltage is the same on both side, so too must be the electric field, and so

$$\vec{E}_{vac} = \vec{E} = \frac{\vec{D}}{\varepsilon} \quad E_{vac} = -\frac{V_1}{d}\hat{z} \Rightarrow \sigma_A = D = \varepsilon E = \frac{\varepsilon V_1}{d} = \frac{\varepsilon V}{\varepsilon_r d} = \varepsilon_0 \frac{V}{d}$$

While on the vacuum side $\sigma_B = \varepsilon_0 E_{vac} = \frac{\varepsilon_0 V_1}{d}$

so $Q_1 = A(\sigma_A + \sigma_B) = \frac{AV_1}{d}(\varepsilon + \varepsilon_0) = \frac{AV_1}{d}(\varepsilon_r\varepsilon_0 + \varepsilon_0) = \frac{A\varepsilon_0 V_1}{d}(\varepsilon_r + 1)$

while $Q = A2\sigma_B = \frac{2A\varepsilon_0 V}{d}$

so $\frac{Q_1}{Q} = \frac{V_1}{2V}(\varepsilon_r + 1)$ and $\frac{C_1}{C} = \frac{Q_1}{Q}\frac{V}{V_1} = \frac{1}{2}\frac{V_1}{V}(\varepsilon_r + 1)\frac{V}{V_1} = \frac{\varepsilon_r + 1}{2}$

obviously $\vec{P}_B = \vec{0}$

while $\vec{P}_A = \varepsilon_0 \chi_e \vec{E} = \varepsilon_0(\varepsilon_r - 1)\vec{E} = \varepsilon_0(\varepsilon_r - 1)\left(\frac{-V_1}{d}\hat{z}\right) = \varepsilon_0(1 - \varepsilon_r)\frac{V_1}{d}\hat{z}$

and $(\sigma_A)_b = \vec{P}_A \cdot \hat{n} = \pm\varepsilon_0(1 - \varepsilon_r)\frac{V_1}{d}$ where + is on the top surface wile − is on the bottom surface