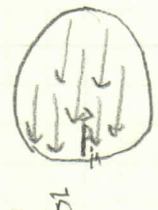
Kinkle, Zachary | EM Griffiths | 09/17/18 | 1/1

4.16 $\vec{D}_0 = \varepsilon_0 \vec{E}_0 + \vec{P}$

a)



$\uparrow \vec{P}$ The field of a uniformly polarized sphere is $\vec{E} = -\frac{1}{3\varepsilon_0}\vec{P}$. So $\vec{E}_{in} = -\frac{1}{3\varepsilon_0}\vec{P}_{in} = \frac{1}{3\varepsilon_0}\vec{P}$

$\vec{P}_{in} = -\vec{P}$ The field inside the cavity is then,

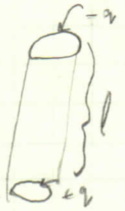
$\vec{E}_{net} = \vec{E}_{in} + \vec{E}_0 = \frac{1}{3\varepsilon_0}\vec{P} + \vec{E}_0 = \frac{1}{3\varepsilon_0}\vec{P} + \frac{1}{\varepsilon_0}(\vec{D}_0 - \vec{P}) = -\frac{2}{3\varepsilon_0}\vec{P} + \frac{D_0}{\varepsilon_0}$

To find $\vec{D}$ inside the cavity, use the Definition of $\vec{D}$. $\vec{D} = \varepsilon_0 \vec{E}_{net} + \vec{P}_{net}$ zero inside the cavity

$\vec{D} = \varepsilon_0 \vec{E} + \vec{0} = \varepsilon_0\left(-\frac{2}{3\varepsilon_0}\vec{P} + \frac{\vec{D}_0}{\varepsilon_0}\right) = \vec{D}_0 - \frac{2}{3}\vec{P}$

b) First we must determine the Electric field of a uniformly polarized cylinder:

Since it is thin, we need only consider points on it's axis. This boils down to the electric field of a disk.



Using the solution to problem 2.6: $\vec{E} = \frac{\sigma_b}{2\varepsilon_0}\left[\frac{\sqrt{a^2+z^2} + z}{\sqrt{a^2+z^2}}\right]\hat{z}$

Setting the reference at the bottom disk, the field due to the top disc is found using the substitution $z \to z - l$. However, since $l \gg a$, the field reduces to that of a point charge: $\vec{E} = \frac{\pi a^2 \sigma_b}{z^2}\hat{z}$

So the field is $\vec{E} = \pi a^2 \sigma_b\left(\frac{1}{z^2} - \frac{1}{(z-l)^2}\right) = \pi a^2 P_{in}\left(\frac{1}{z^2} - \frac{1}{(z-l)^2}\right)$ i.e.

Thus, $\vec{E}_{net} = -\pi a^2 P_{in}\left(\frac{1}{z^2} - \frac{1}{(z-l)^2}\right) + \vec{E}_0$. However, since a is very, very small

$a^2 \approx 0$, so $\vec{E}_{net} = \vec{E}_0 = \frac{1}{\varepsilon_0}(\vec{D}_0 - \vec{P})$

Then $\vec{D}_{net} = \varepsilon_0 \vec{E}_{net} + 0 = \vec{D}_0 - \vec{P}$.

C) This time we can consider the cylinder so thin that $\vec{E}$ is uniform inside the wafer, and approximate it as an infinite sheet:

$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$. The Top surface gives $\vec{E} = \frac{P}{2\varepsilon_0}$, and the bottom surface gives

$\vec{E} = \frac{P}{2\varepsilon_0}$, so $\vec{E}_{net} = \frac{\vec{P}}{\varepsilon_0} + \vec{E}_0 = \frac{\vec{P}}{\varepsilon_0} + \frac{1}{\varepsilon_0}(\vec{D}_0 - \vec{P}) = \frac{\vec{D}_0}{\varepsilon_0}$

Thus, the displacement inside is $\vec{D}_{net} = \varepsilon_0 \vec{E}_{net} + 0 = \vec{D}_0$