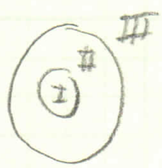
4.15

a) $\sigma_a = \vec{P}(a\hat{r}) \cdot \hat{n} = \frac{k}{a}\hat{r} \cdot (-\hat{r}) = -\frac{k}{a}$; $\sigma_b = \vec{P}(b\hat{r}) \cdot \hat{n} = \frac{k}{b}\hat{r} \cdot (\hat{r}) = \frac{k}{b}$

$q_a = 4\pi a^2 \sigma_a = -4\pi a k, \quad q_b = 4\pi b^2 \sigma_b = 4\pi b k$

$\rho = -\vec{\nabla} \cdot \vec{P}(\vec{r}) = \frac{-1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{k}{r}\right) = -\frac{k}{r^2}$

$q_{a\text{-}b} = \int \rho \, d\tau = -k \int \sin\theta \, dr \, d\theta \, d\phi = -4\pi k(b-a)$



$\vec{E}_{I} = 0$

$4\pi r^2 E_{II} = \frac{q_a + q_{a\text{-}r}}{\varepsilon_0} = \frac{-4\pi k a - 4\pi k(r-a)}{\varepsilon_0} = \frac{-4\pi k r}{\varepsilon_0}$

$\Rightarrow \vec{E}_{II} = -\frac{k}{\varepsilon_0 r}\hat{r}$

$4\pi r^2 E_{III} = \frac{q_a + q_b + q_{a\text{-}b}}{\varepsilon_0} = \frac{-4\pi k a + 4\pi k b - 4\pi k(b-a)}{\varepsilon_0} = 0$

$\Rightarrow \vec{E}_{III} = 0$

b) $4\pi r^2 D = 0 \Rightarrow \vec{D} = \varepsilon_0 \vec{E} + \vec{P} = 0 \Rightarrow \vec{E} = -\vec{P}/\varepsilon_0$

$\Rightarrow \vec{E}_{I} = \frac{-\vec{P}_{I}}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$

$\vec{E}_{II} = \frac{-\vec{P}_{II}}{\varepsilon_0} = -\frac{k}{\varepsilon_0 r}\hat{r}$

$\vec{E}_{III} = 0$