4.12

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\vec{P}(\vec{r})\cdot\hat{\mathscr{r}}}{\mathscr{r}^2}d\tau' \qquad \vec{P}(\vec{r}) = \vec{P}_0$$

$$V(\vec{r}) = \frac{\vec{P}_0}{4\pi\varepsilon_0}\cdot\int\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}d\tau'$$

notice, the substitution $\vec{P}_0\cdot \rightarrow \rho_0$ makes this integral equivalent to that of a uniformly charged solid sphere. Thus we can use the field eq. making the substitution $\rho_0 \rightarrow \vec{P}_0\cdot$

$$\Rightarrow V(\vec{r}) = \begin{cases} \frac{r}{3\varepsilon_0}\vec{P}_0\cdot\hat{z} & \text{for } r<R \\ \frac{R^3}{3\varepsilon_0 r^2}\vec{P}_0\cdot\hat{z} & \text{for } r\geq R \end{cases}$$

$$\Rightarrow V(\vec{r}) = \begin{cases} P_0\frac{r\cos\theta}{3\varepsilon_0} = P_0\frac{z}{3\varepsilon_0} & \text{for } r<R \\ P_0\frac{R^3\cos\theta}{3\varepsilon_0 r^2} & \text{for } r\geq R \end{cases}$$