

$$4.10 \text{ a) } \vec{P}(\vec{r}) = k r \hat{r} = k(x\hat{x} + y\hat{y} + z\hat{z})$$

$$\sigma_b = \vec{P}(\vec{r}) \cdot \hat{n} = \vec{P}(R\hat{r}) \cdot \hat{r} = kR\hat{r} \cdot \hat{r} = kR$$

$$\rho_b(\vec{r}) = -\vec{\nabla} \cdot \vec{P}(\vec{r}) = -k \vec{\nabla} \cdot (x\hat{x} + y\hat{y} + z\hat{z}) = -3k$$

b) The fields can be found by treating this as a problem of finding the electric field of a constant surface charge and constant volume charge. These problems have both been solved already previously in Chapter 2. The answers are

$$\vec{E}_\sigma = \begin{cases} 0 & \text{for } r < R \\ \frac{\sigma}{\epsilon_0} \left(\frac{R}{r}\right)^2 \hat{r} & \text{for } r > R \end{cases} = \begin{cases} 0 & \text{for } r < R \\ \frac{kR}{\epsilon_0} \left(\frac{R}{r}\right)^2 \hat{r} & \text{for } r > R \end{cases}$$

$$\vec{E}_\rho = \begin{cases} \frac{\rho r}{3\epsilon_0} \hat{r} & \text{for } r < R \\ \frac{\rho R^3}{3\epsilon_0 r^2} \hat{r} & \text{for } r > R \end{cases} = \begin{cases} -\frac{k}{\epsilon_0} \vec{r} & \text{for } r < R \\ -\frac{kR^3}{\epsilon_0 r^2} \hat{r} & \end{cases}$$

$$\text{Thus } \vec{E}_{\vec{P}}(\vec{r}) = \begin{cases} -\frac{k}{\epsilon_0} \vec{r} & \text{for } r < R \\ 0 & \text{for } r > R \end{cases}$$