3.30/3.31 | Kibler Zachary | EM Griffiths | 07/22/18 | 1/1

3.30

a) $\vec{p} = \int \vec{r}\,'\sigma(\vec{r}\,')\,da'$; $\vec{r}\,' = R\hat{r}' = R\sin\theta\cos\phi\,\hat{x} + R\sin\theta\sin\phi\,\hat{y} + R\cos\theta\,\hat{z}$

$\sigma(\vec{r}\,') = k\cos\theta$; $da' = R^2\sin\theta\,d\theta\,d\phi$

$$\vec{p} = \int R\hat{r}'k\cos\theta\,R^2\sin\theta\,d\theta\,d\phi = R^3k\int \sin^2\theta\cos\theta\cos\phi\,\hat{x}\,d\theta\,d\phi + R^3k\int \sin^2\theta\cos\theta\sin\phi\,\hat{y}\,d\theta\,d\phi + R^3k\int \cos^2\theta\sin\theta\,d\theta\,d\phi$$

$\int_0^\pi \sin^2\theta\cos\theta\,d\theta = 0$ (see problem 3.27), so the $\hat{x}$ & $\hat{y}$ components are zero

let $-\cos\theta = u$, $-\sin\theta\,d\theta = du$

$\int_0^\pi \cos^2\theta\sin\theta\,d\theta = -\int_{1}^{-1} u^2\,du = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$, so $\vec{p} = \frac{4}{3}\pi R^3 k\,\hat{z}$

b) $V_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\,\frac{4}{3}\pi R^3 k\,\hat{z}\cdot\hat{r} = \frac{R^3 k\cos\theta}{3\varepsilon_0 r^2}$. This is exactly the potential outside of the sphere! So evidently all other terms in the multipole expansion must be zero.

3.31 i) $Q = 2q$ for a), b), and c).

ii) a) $\vec{p} = 3qa\,\hat{z}$

b) $\vec{p} = -q(-a\hat{z}) = qa\,\hat{z}$

c) $\vec{p} = 3qa\,\hat{y}$

iii) $V(\vec{r}) \cong V_{mon}(\vec{r}) + V_{dip}(\vec{r})$. For a), b), and c) $V_{mon}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r}$

a) $V_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{3qa\cos\theta}{r^2}$; $V(\vec{r}) \cong \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}\left(2 + \frac{3a\cos\theta}{r}\right)$

b) $V_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{qa\cos\theta}{r^2}$; $V(\vec{r}) \cong \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}\left(2 + \frac{a\cos\theta}{r}\right)$

c) $V_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{3qa\sin\theta\sin\phi}{r^2}$; $V(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}\left(2 + \frac{3a\sin\theta\sin\phi}{r^2}\right)$