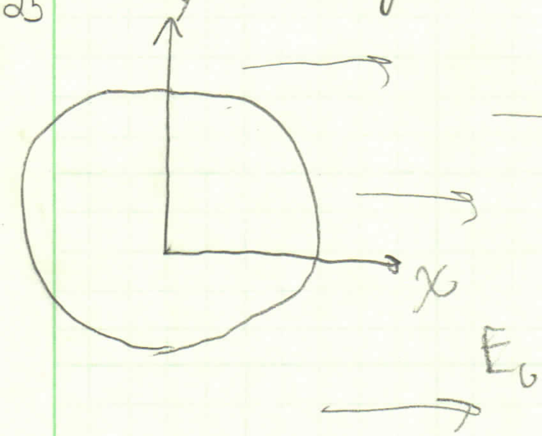
Hinkle, Zachary | EM Griffiths | 07/16/18 | 1/1

3.25



$$V(s,\phi) = A_0 \ln(s) + B_0 + \sum_{k=1}^{\infty}\left[(A_k s^k + B_k s^{-k})(C_k \sin(k\phi) + D_k \cos(k\phi))\right]$$

the pipe is a conductor and hence an equipotential, so

$V(R,\phi) = 0$. Also, since $s \to \infty$ $V \to -E_0 x + C = -E_0 s\cos\phi + C$,

at $\phi = \frac{\pi}{2}$, at large s $V \to -E s\cos(\frac{\pi}{2}) + C = 0 + C$.

The yz-plane is an equipotential plane, and we know at the point $(0, R, 0)$, $V = 0$. Since this is on the yz-plane, $C = 0$. So $s \to \infty$ $V \to -E_0 s\cos\phi$.

The BC's then are

$$V = \begin{cases} 0 & \text{at } s = R, \quad (i) \\ -E_0 s\cos\phi & \text{at } s >> R. \quad (ii) \end{cases}$$

From (ii)

$$V(s,\phi) = A_0 \ln(s) + B_0 + \sum_{k=1}^{\infty} A_k s^k (C_k \sin(k\phi) + D_k \cos(k\phi)) = -E_0 s\cos\phi$$
at large s.

So Evidently $A_0 = B_0 = 0$. $A_k C_k = 0$ for all k. $A_{k>1} D_{k>1} = 0$. $A_1 D_1 = -E_0$

From (i)

$$V(R,\phi) = 0 = -E_0 R\cos\phi + \sum_{k=1}^{\infty} B_k R^{-k} D_k \cos(k\phi) \Rightarrow B_{k\neq1} D_{k\neq1} = 0 \Rightarrow E_0 R = \frac{B_k D_k}{R}$$

$$\Rightarrow B_k D_k = E_0 R^2 \Rightarrow V(s,\phi) = (A_1 s^1 + B_1 s^{-1})(D_1 \cos\phi) = -E_0\left(s - \frac{R^2}{s}\right)\cos\phi$$