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3.13 Much of the heavy lifting to this problem has already been done. The same general form solution may be used so long as the coefficients are altered appropriately.

New BCs:

(i) $V=0$ at $y=0$

(ii) $V=0$ at $y=a$

(iii) $V=\begin{cases} V_0 & \text{at } x=0 \ \& \ y\in[0, a/2] \\ -V_0 & \text{at } x=0 \ \& \ y\in(a/2, a] \end{cases}$

(iv) $V\to 0$ as $x\to\infty$

Then as before

$$V(x,y) = \sum_{n=1}^{\infty} C_n e^{-n\pi x/a} \sin(n\pi y/a)$$

and

$$V(0,y) = \sum_{n=1}^{\infty} C_n \sin(n\pi y/a) = \begin{cases} V_0 & \text{for } y\in[0, a/2] \\ -V_0 & \text{for } y\in(a/2, a] \end{cases}$$

then

$$\frac{a C_n}{2} = V_0\int_0^{a/2} \sin(n\pi y/a)\, dy - V_0\int_{a/2}^{a} \sin(\pi n y/a)\, dy$$

so

$$\left(\frac{a}{2}\right)\left(\frac{n\pi}{a}\right)\frac{C_n}{V_0} = \left[\cos\left(\frac{n\pi y}{a}\right)\right]_{a/2}^{0} + \left[\cos\left(\frac{\pi n y}{a}\right)\right]_{a/2}^{a} = \cos(0) - \cos\left(\frac{n\pi}{2}\right) + \cos(n\pi) - \cos\left(\frac{n\pi}{2}\right)$$

$$C_n = \frac{2V_0}{n\pi}\left[1+\cos(n\pi)\right] = \begin{cases} \frac{4V_0}{n\pi} & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd} \end{cases}$$

so

$$V(x,y) = \frac{4V_0}{\pi}\sum_{n=2,4,6,\ldots}^{\infty} \frac{1}{n} e^{-n\pi x/a} \sin(n\pi y/a)$$