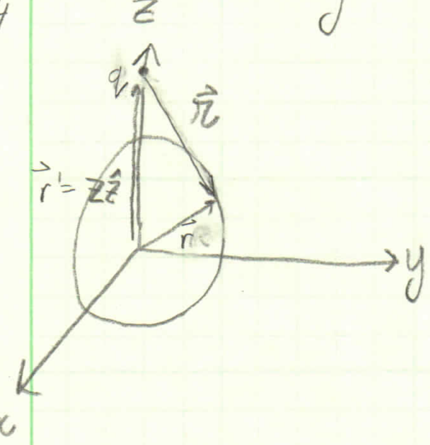
3.4



$$\vec{\mathscr{r}} = \vec{r} - \vec{r}' = R\hat{r} - Z\hat{z}$$

$$E(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\vec{\mathscr{r}} = R\sin\theta\cos\phi\,\hat{x} + R\sin\theta\sin\phi\,\hat{y} + (R\cos\theta - Z)\hat{z}$$

$$\mathscr{r}^2 = R^2\sin^2\theta(\cos^2\phi + \sin^2\phi) + R^2\cos^2\theta - 2ZR\cos\theta + Z^2$$

$$\mathscr{r} = \sqrt{Z^2 + R^2 - 2ZR\cos\theta}$$

$$\vec{E}_{ave} = \frac{1}{4\pi R^2}\frac{q}{4\pi\varepsilon_0}\int \frac{R\sin\theta\cos\phi\,\hat{x} + R\sin\theta\sin\phi\,\hat{y} + (R\cos\theta - Z)\hat{z}}{\sqrt{Z^2 + R^2 - 2ZR\cos\theta}} R^2\sin\theta\, d\theta\, d\phi$$

recognizing the similarity to problem 2.7, with $\sigma = \frac{q}{4\pi R^2}$, we have

$$\vec{E}_{2.7} = \frac{q}{4\pi R^2}\frac{1}{4\pi\varepsilon_0}\int \frac{(Z - R\cos\theta)\hat{z} - R\sin\theta\cos\phi\,\hat{x} - R\sin\theta\sin\phi\,\hat{y}}{\sqrt{Z^2 + R^2 - 2ZR\cos\theta}} R^2\sin\theta\, d\theta\, d\phi$$

so $\vec{E}_{ave} = -\vec{E}_{2.7} \Rightarrow E_{ave} = E_{2.7} = \frac{1q}{4\pi\varepsilon_0 Z^2}$

b) now we want to know for $Z < R$. So,

$\vec{E}_{ave} = -\vec{E}_{2.7} = 0$ for $Z < R$.