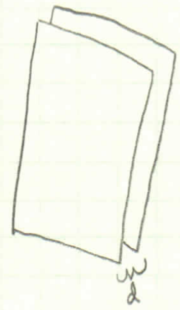
2.41 & 2.42 | Vinkle, Zachary | EM Griffiths | 07/10/18 | 1/1

2.41

a) $\sigma = \dfrac{Q}{A}$ $\qquad \vec{E}_{between} = 0$



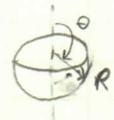
For either plate

$$\vec{E}_{outside} - \vec{E}_{between} = \frac{\sigma}{\varepsilon_0}\hat{n} \Rightarrow \vec{E}_{outside} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

$$\vec{f} = \frac{1}{2}\sigma\left(E_{outside} + E_{between}\right) = \frac{\sigma^2}{2\varepsilon_0}\hat{n} = \frac{Q^2}{2\varepsilon_0 A^2}\hat{n}$$

2.42

The direction of the force can be determined as follows:



$\sigma = \dfrac{Q}{4\pi R^2}$

$$\vec{A} = \int d\vec{a} = \int R^2 \sin\theta \, d\theta \, d\phi \, \hat{r} = \int R^2 \sin\theta \, d\phi \, d\theta \left(\sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}\right)$$

Since $\int_0^{2\pi} \cos\phi \, d\phi = \int_0^{2\pi} \sin\phi \, d\phi = 0$, the $\hat{x}$ and $\hat{y}$ components are 0.

$$\vec{A} = R^2 \int_0^{2\pi}\int_0^{\pi/2} \sin\theta\cos\theta \, d\theta \, d\phi \, \hat{z} = 2\pi R^2 \left[\frac{1}{2}\sin^2\theta\right]_0^{\pi/2} \hat{z} = \pi R^2 \hat{z}$$

notice this integral is different from its non-vector cousin: $A = \int da = 2\pi R^2$

$\Rightarrow \hat{A} = \hat{z}$ this is the direction of the force

$$F = \int_S \vec{f} \cdot d\vec{a} \qquad d\vec{a} = R^2 \sin\theta \, d\theta \, d\phi \, \hat{n} \qquad \vec{f} = \frac{\sigma^2}{2\varepsilon_0}\hat{n}$$

$$F = \int_S \frac{\sigma^2}{2\varepsilon_0} R^2 \sin\theta \, d\theta \, d\phi = \frac{\pi\sigma^2 R^2}{2\varepsilon_0}\int_0^{\frac{\pi}{2}} \sin\theta \, d\theta = \frac{-\pi\sigma^2 R^2}{2\varepsilon_0}\cos\theta \Big|_0^{\frac{\pi}{2}} = \frac{\pi\sigma^2 R^2}{2\varepsilon_0} = \frac{\pi Q^2 R^2}{2(4\pi R^2)^2 \varepsilon_0}$$

$$F = \frac{Q^2}{32\pi\varepsilon_0 R^2} \Rightarrow \vec{F} = \frac{Q^2}{32\pi\varepsilon_0 R^2}\hat{z}$$

Alternatively, we can think of $\vec{A} = \pi R^2 \hat{z}$ as the area "seen" by the pressure $\frac{\varepsilon_0}{2}E^2$. After all, πR^2 is simply the projection of the hemisphere onto the z-axis. Then

$$\vec{F} = P\vec{A} = \frac{\varepsilon_0}{2}\left(\frac{\sigma^2}{\varepsilon_0^2}\right)\pi R^2 \hat{z} = \frac{Q^2 \pi R^2}{2(4\pi R^2)^2 \varepsilon_0}\hat{z} = \frac{Q^2}{32\pi\varepsilon_0 R^2}\hat{z}$$