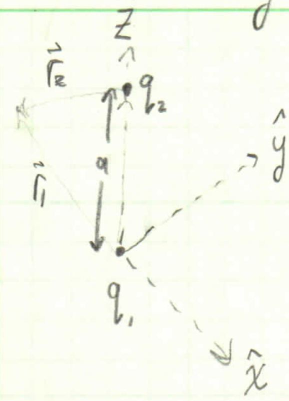
2.37



$$E_1 = \frac{kq_1}{r_1^2}\hat{r}_1$$

$$\vec{r}_1 = r\hat{r}$$
$$\vec{r}_1 = a\hat{z} + \vec{r}_2$$
$$\vec{r}_2 = \vec{r}_1 - a\hat{z}$$

$$E_2 = \frac{kq_2}{r_2^2}\hat{r}_2 \qquad r_2^2 = \vec{r}_2\cdot\vec{r}_2 = (r\hat{r}-a\hat{z})\cdot(r\hat{r}-a\hat{z})$$

$$E_2 = kq_2\frac{r\hat{r}-a\hat{z}}{(r^2+a^2-2ra\cos\theta)^{3/2}} \qquad = r^2+a^2 - ra\hat{r}\cdot\hat{z} - ar\hat{z}\cdot\hat{r}$$

$$\hat{r}\cdot\hat{z} = \hat{z}\cdot\hat{r} = \cos\theta$$

let $E^2 = \vec{E}_1\cdot\vec{E}_2$

$$r_2^2 = \vec{r}_2\cdot\vec{r}_2 = r^2+a^2-2ra\cos\theta$$

$$E^2 = \frac{k^2 q_1 q_2}{r(r^2-2ra\cos\theta+a^2)}$$

$$\hat{r}_2 = \frac{r\hat{r}-a\hat{z}}{\sqrt{r^2+a^2-2ra\cos\theta}}$$

$$E^2 d\tau = k^2 q_1 q_2 \frac{r\sin\theta\, dr\, d\phi\, d\theta}{(r^2-2ra\cos\theta+a^2)^{3/2}}$$

let $u = r^2 - 2ra\cos\theta + a^2 \Rightarrow du = 2ra\sin\theta\, d\theta$

$$k^2q_1q_2\int_0^\pi \frac{r\sin\theta\, d\theta}{(r^2-2ra\cos\theta+a^2)^{3/2}} = \frac{k^2q_1q_2}{2a}\int_{r^2-2ra+a^2=(r-a)^2}^{r^2+2ra+a^2=(r+a)^2} u^{-3/2}\,du = -\frac{k^2q_1q_2}{a\sqrt{u}}\Bigg|_{(r-a)^2}^{(r+a^2)}$$

$$= \frac{k^2q_1q_2}{2a\sqrt{(r-a)^2}} + \frac{k^2q_1q_2}{2a(r+a)} = \begin{cases} \frac{k^2q_1q_2}{2a}\left(\frac{1}{(a-r)} + \frac{1}{a+r}\right); & r\in[0,a) \\ \frac{k^2q_1q_2}{2a}\left(\frac{1}{r-a} + \frac{1}{r+a}\right), & r\in(a,\infty) \end{cases}$$

$$\int E^2 d\tau = \frac{\pi\varepsilon_0 q_1q_2}{a}\left[\int_0^a \frac{1}{a-r} + \int_a^\infty \frac{1}{r-a} + \int_0^\infty \frac{1}{r+a}\right]$$

let $u = a-r \Rightarrow du = -dr$
let $u' = r-a \Rightarrow du = dr$
let $u'' = r+a \Rightarrow du = dr$

$$= \frac{\pi\varepsilon_0 q_1q_2}{a}\left(-\int_a^0 u^{-1}du + \int_0^\infty (u')^{-1}du + \int_a^\infty (u'')^{-1}du\right)$$

$$= \frac{\pi\varepsilon_0 q_1q_2}{a}\left([\ln u]_0^a + [\ln u']_0^\infty + [\ln u'']_0^\infty\right) \quad ??$$