2.36 a) $W = \frac{\varepsilon_0}{2}\int_{\mathcal{V}} E^2 \, d\tau$ over all space

$$E = \begin{cases} 0 & \text{for } r < R \\ \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r} & \text{for } r > R \end{cases}$$

$$E_1 = \begin{cases} 0 & \text{for } r < a \\ \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r} & \text{for } r > a \end{cases}$$

$$E_2 = \begin{cases} 0 & \text{for } r < b \\ \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r} & \text{for } r > b \end{cases}$$

with $a < b$ then,

$$E = E_1 + E_2 = \begin{cases} 0 & \text{for } r < a \\ \frac{q_1}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r} & \text{for } a < r < b \\ \frac{q_a + q_b}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r} & \text{for } r > b \end{cases}$$

let $k = \frac{1}{4\pi\varepsilon_0}$

$$E^2 = \begin{cases} 0 & \in r < a \\ \frac{k^2 q_a^2}{r^4} & \in a < r < b \\ \frac{k^2 (q_a + q_b)^2}{r^4} & \in r > b \end{cases}$$

$$W = \frac{\varepsilon_0}{2}\int_{\mathcal{V}} E^2 \, d\tau = \frac{\varepsilon_0}{2}\int_{\mathcal{V}} 0 \, d\tau + \frac{\varepsilon_0}{2}\int_{\mathcal{V}} \frac{k^2 q_a^2}{r^4} \, d\tau + \frac{\varepsilon_0}{2}\int_{\mathcal{V}} \frac{k^2 (q_a^2 + q_b^2)}{r^4} \, d\tau$$

$$W = \frac{4\pi\varepsilon_0 k^2 q_a^2}{2}\int_a^b \frac{dr}{r^2} + \frac{4\pi\varepsilon_0 k^2 (q_a + q_b)^2}{2}\int_b^\infty \frac{dr}{r^2}$$

$$W = -\frac{k q_a^2}{2}\left[\frac{1}{b} - \frac{1}{a}\right] - \frac{k(q_a + q_b)^2}{2}\left[\frac{1}{\infty} - \frac{1}{b}\right] = \frac{k q_a^2}{2}\left[\frac{1}{a} - \frac{1}{b}\right] + \frac{k q_a^2}{2}\left[\frac{1}{b}\right] + k q_a q_b \left[\frac{1}{b}\right] + \ldots$$

$$W = \frac{k q_a^2}{2}\left[\frac{1}{a}\right] + k q_a q_b \left[\frac{1}{b}\right] + \frac{k q_B^2}{2}\left[\frac{1}{b}\right] = \frac{k}{2}\left[\frac{b q_a^2 + 2a q_a q_b + a q_b^2}{ab}\right]$$

w/ $q_a = -q_B = q$ $\quad W = \frac{k}{2}\left[\frac{b q^2 - 2a q^2 + a q^2}{ab}\right] = \frac{k q^2}{2}\left[\frac{b - a}{ab}\right]$

$$W = \frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

2.36 b) $W_{sphere} = \frac{q^2}{8\pi\varepsilon_0} \cdot \frac{1}{R} \Rightarrow W_a = \frac{q_a^2}{8\pi\varepsilon_0 a}$; $W_b = \frac{q_b^2}{8\pi\varepsilon_0 b}$

$$W = W_a + W_b + \varepsilon_0 \int_{\mathcal{V}\text{ all space}} \vec{E}_1 \cdot \vec{E}_2 \, d\tau$$

let $E^2 = \vec{E}_1 \cdot \vec{E}_2 = \begin{cases} 0 & \text{for } r < b \\ \frac{k^2 q_a q_b}{r^4} & \text{for } r > b \end{cases}$

$$\varepsilon_0 \int E^2 d\tau = 4\pi\varepsilon_0 k^2 q_a q_b \int_b^\infty r^{-2} dr = k q_a q_b \left[\frac{1}{r}\right]_\infty^b = \frac{k q_a q_b}{b}$$

$$W = \frac{k}{2}\left[\frac{q_a^2}{a} + \frac{q_b^2}{b} + \frac{2 q_a q_b}{b}\right] = \frac{k}{2}\left[\frac{b q_a^2 + 2a q_a q_b + a q_b^2}{ab}\right]$$

w/ $q_a = -q_b = q$ $\quad W = \frac{k}{2}\left[\frac{b q^2 - 2a q^2 + a q^2}{ab}\right] = \frac{k q^2}{2}\left[\frac{b-a}{ab}\right]$

$$W = \frac{q^2}{8\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$