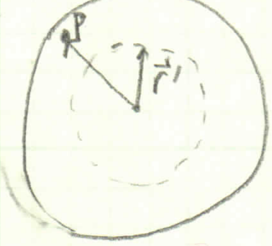
Hinkle, Zachary | EM Griffiths | 06/28/18 | 1

2.21



$$\rho = \frac{q}{\frac{4}{4}\pi R^3} \qquad V(\vec{r}) = \frac{\rho}{4\pi\varepsilon_0}\int_V \frac{d\tau'}{\mathscr{r}}$$

$$\vec{r} = z\hat{z}$$

$$d\tau' = r^2 \sin\theta \, dr\, d\theta\, d\phi$$

let $k = \frac{1}{4\pi\varepsilon_0}$

$$\vec{r}' = r\hat{r}$$

$$\vec{\mathscr{r}} = z\hat{z} - r\hat{r}$$

from 2.26: $|\vec{\mathscr{r}}| = \mathscr{r} = \sqrt{r^2 + z^2 - rz\cos\theta}$

From example 8

$$\frac{V(\vec{r})}{\rho k} = \int_V \frac{r^2 \sin\theta \, dr\, d\theta\, d\phi}{\sqrt{r^2+z^2-2rz\cos\theta}} = 4\pi\int_0^z r\,dr + \frac{4\pi}{z}\int_z^R r^2\,dr = 2\pi z^2 + \frac{4}{3}\frac{\pi}{z}\left(R^3 - z^3\right)$$

$$= \frac{6\pi z^3}{3z} + \frac{4\pi\left(R^3 - z^3\right)}{3z} = \frac{2\pi}{3z}\left(2R^3 + 2\pi z^3\right)$$

$$\frac{V(\vec{r})}{\rho} = \frac{2\pi}{3z}\,\frac{1}{4\pi\varepsilon_0}\left(\ldots\right) = \frac{R^3}{6z\varepsilon_0}\left(2 - \left(\frac{z}{R}\right)^3\right)$$

$$V(\vec{r}) = \frac{3q}{8\pi\varepsilon_0 z}\left(2 - \left(\frac{z}{R}\right)^3\right) \; ?$$