2.22 & 2.23 $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 s}\hat{s}$

$$V = -\int_{s_0}^{s} \vec{E}\cdot d\vec{l} = -\frac{\lambda}{2\pi\varepsilon_0}\int_{s_0}^{s} s^{-1}\,ds = -\frac{\lambda}{2\pi\varepsilon_0}\ln(s)\Big|_{s_0}^{s} = \frac{\lambda}{2\pi\varepsilon_0}\left[\ln(s_0) - \ln(s)\right]$$

$$\nabla T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$-\nabla V = \frac{\lambda}{2\pi\varepsilon_0 s}\hat{s}$$

2.23 $V(r) = -\int_{\infty}^{b} \vec{E}\cdot d\vec{l}$

$$\vec{E} = \begin{cases} 0 & \text{for } r<a \\ k\frac{r-a}{\varepsilon_0 r^2}\hat{r} & \text{for } a\le r\le b \\ k\frac{b-a}{\varepsilon_0 r^2}\hat{r} & \text{for } r>b \end{cases}$$

$$V(r) = -\frac{k}{\varepsilon_0}(b-a)\int_{\infty}^{b} r^{-2}\,dr - \frac{k}{\varepsilon_0}\int_{b}^{a}\frac{dr}{r} + \frac{ak}{\varepsilon_0}\int_{b}^{a} r^{-2}\,dr - \cancel{\int_{a}^{0} 0\,dr}$$

$$-\int r^{-2}\,dr = \frac{1}{r}$$

$$V(r) = \frac{k}{\varepsilon_0}\frac{(b-a)}{r}\Big|_{\infty}^{b} - \frac{k}{\varepsilon_0}\ln(r)\Big|_{b}^{a} - \frac{ak}{\varepsilon_0 r}\Big|_{b}^{a} = \frac{k}{\varepsilon_0}\frac{b-a}{b} - \frac{k}{\varepsilon_0}(\ln(a)-\ln(b)) - \frac{k}{\varepsilon_0} + \frac{k}{\varepsilon_0}\frac{a}{b}$$

$$= \frac{k}{\varepsilon_0} - \frac{k}{\varepsilon_0}\frac{a}{b} - \frac{k}{\varepsilon_0}\left[\ln(a)-\ln(b)\right] - \frac{k}{\varepsilon_0} + \frac{k}{\varepsilon_0}\frac{a}{b} = -\frac{k}{\varepsilon_0}\left[\ln(a)-\ln(b)\right]$$

$$V(r) = \frac{k}{\varepsilon_0}\left[\ln(b)-\ln(a)\right] = \frac{k}{\varepsilon_0}\ln\left(\frac{b}{a}\right)$$