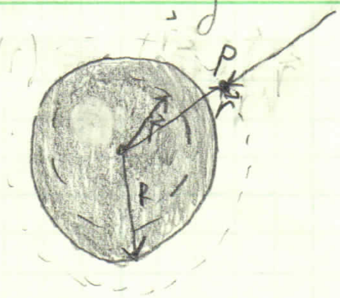
Hinkle, Zachary | EM Griffiths | 06/06/18 | 1/1

2.21



Case 1: Outside Sphere:

$Q_{enc} = q$ so

$$|\vec{E}_1| \oint_s d\vec{A} = |\vec{E}|\, 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow \vec{E}_1 = \frac{q}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}$$

$$V(\vec{r}) = -\int_\infty^r \vec{E}\cdot d\vec{l} = -\frac{q}{4\pi\varepsilon_0}\int_\infty^r r^{-2}\,dr = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\Big|_\infty^r = \frac{q}{4\pi\varepsilon_0}\frac{1}{r} - 0$$

Case 2: Inside Sphere:

$$Q_{enc} = q\left(\frac{r}{R}\right)^3$$

so

$$|\vec{E}_2|\, 4\pi r^2 = \frac{q}{\varepsilon_0}\left(\frac{r}{R}\right)^3 = \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}\, r\,\hat{r}$$

$$V(\vec{r}) = -\int_\infty^r \vec{E}\cdot d\vec{l} = -\frac{q}{4\pi\varepsilon_0}\int_\infty^R r^{-2}\,dr - \frac{q}{4\pi\varepsilon_0}\frac{1}{R^3}\int_R^r r\,dr$$

$$= \frac{q}{4\pi\varepsilon_0 R} - \frac{q}{4\pi\varepsilon_0}\frac{1}{R^3}\left[\tfrac{1}{2}r^2\right]_R^r = \frac{q}{4\pi\varepsilon_0 R} - \frac{q}{8\pi\varepsilon_0}\frac{1}{R^3}\left[r^2 - R^2\right]$$

$$= \frac{3q}{8\pi\varepsilon_0 R} - \frac{q}{8\pi\varepsilon_0 R}\left(\frac{r}{R}\right)^2 = \frac{\frac{1}{2}q}{4\pi\varepsilon_0 R}\left[3 - \left(\frac{r}{R}\right)^2\right]$$