2.20

$$\vec{\nabla}\times\vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{z}$$

$$\frac{\vec{\nabla}\times\vec{E}_a}{k} = (x-2y)\hat{x} + (0-3z)\hat{y} + (0-x)\hat{z} \neq 0$$

$$\frac{\vec{\nabla}\times\vec{E}_b}{k} = (2z-2z)\hat{x} + (0-0)\hat{y} + (2y-2y)\hat{z} = 0$$

$$V_b(\vec{r}) = -\int_{\text{origin}}^{\vec{r}_0} \vec{E}_b\cdot d\vec{l} \qquad \text{let } \vec{r}_0 = (x_0, y_0, z_0)$$

$$V_b(\vec{r}_0) = -\int_{(0,0,0)}^{(x_0,0,0)} \vec{E}_b\cdot d\vec{l}_1 - \int_{(x_0,0,0)}^{(x_0,y_0,0)} \vec{E}_b\cdot d\vec{l}_2 - \int_{(x_0,y_0,0)}^{(x_0,y_0,z_0)} \vec{E}_b\cdot d\vec{l}_3$$

$$d\vec{l}_1 = dx\,\hat{x}, \quad d\vec{l}_2 = dy\,\hat{y}, \quad d\vec{l}_3 = dz\,\hat{z}$$

$$E_b\cdot\hat{x} = ky^2; \quad E_b\cdot\hat{y} = (2xy+z^2)k, \quad E_b\cdot\hat{z} = (2yz)k$$

$$-\int_{(0,0,0)}^{(x_0,0,0)} \vec{E}_b\cdot d\vec{l}_1 = -k\int_{(0,0,0)}^{(x_0,0,0)} y^2\,dx = 0$$

$$-\int_{(x_0,0,0)}^{(x_0,y_0,0)} \vec{E}_b\cdot d\vec{l}_2 = -k\int_{(x_0,0,0)}^{(x_0,y_0,0)} (2xy+z^2)\,dy = -kx_0y_0^2$$

$$-\int_{(x_0,y_0,0)}^{(x_0,y_0,z_0)} \vec{E}_b\cdot d\vec{l}_3 = -k\int_{(x_0,y_0,0)}^{(x_0,y_0,z_0)} 2yz\,dz = -ky_0z_0^2$$

$$V_b(\vec{r}_0) = -k(x_0y_0^2 + y_0z_0^2) \quad \text{so} \quad V_b(\vec{r}) = -k(xy^2+yz^2)$$

$$-\vec{\nabla}V_b(\vec{r}) = k\left(y^2\hat{x} + (2xy+z^2)\hat{y} + (2yz)\hat{z}\right)$$

2.20 as a bonus, I'll use a 2nd path to check the path independence of V

$$V_b(\vec{r}_0) = -\int_{(0,0,0)}^{(0,0,z_0)} \vec{E}_b \cdot d\vec{l}_3 - \int_{(0,0,z_0)}^{(0,y_0,z_0)} \vec{E}_b \cdot d\vec{l}_2 - \int_{(0,y_0,z_0)}^{(x_0,y_0,z_0)} \vec{E}_b \cdot d\vec{l}_1$$

$$-\int_{(0,0,0)}^{(0,0,z_0)} \vec{E}_b \cdot d\vec{l}_3 = -k\int_{(0,0,0)}^{(0,0,z_0)} 2yz\,dz = 0$$

$$-\int_{(0,0,z_0)}^{(0,y_0,z_0)} \vec{E}_b \cdot d\vec{l}_2 = -k\int_{(0,0,z_0)}^{(0,y_0,z_0)} (2xy+z^2)\,dy = -k\,y_0 z_0^2$$

$$-\int_{(0,y_0,z_0)}^{(x_0,y_0,z_0)} \vec{E}_b \cdot d\vec{l}_1 = -k\int_{(0,y_0,z_0)}^{(x_0,y_0,z_0)} y^2\,dx = -k\,x_0 y_0^2$$

so $V_b(\vec{r}) = -k(xy^2 + yz^2)$