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2.19 $$\vec{\nabla}\times\vec{V} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta}$$
$$+\frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$E(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\vec{r}\,')}{\mathscr{r}^2}\hat{\mathscr{r}}\,d\tau'$$

$$\cancel{E_\phi = E_\theta = 0;\quad E_r = k\int\frac{\rho(\vec{r}\,')}{\mathscr{r}^2}\hat{\mathscr{r}}\,d\tau' \quad\therefore E_r = E(\vec{r})}$$

$$\therefore\ \vec{\nabla}\times\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\rho(r')\,\vec{\nabla}\times\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)d\tau'$$

for $\vec{\nabla}\times\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)$ $\quad v_\theta = v_\phi = 0;\quad v_r = v_r(\vec{r})$ so $\frac{\partial v_r}{\partial\theta} = \frac{\partial v_r}{\partial\phi} = 0$

So $\vec{\nabla}\times\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = 0$

hence

$\vec{\nabla}\times\vec{E} = 0$