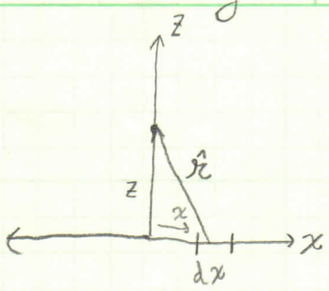
2. Ex 2



$\vec{r} = -x\hat{x} + z\hat{z}$ $\quad r = \sqrt{z^2+x^2}$

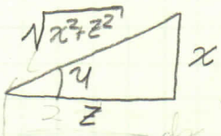
$\hat{r} = \dfrac{-x\hat{x} + z\hat{z}}{\sqrt{z^2+x^2}}$

$E = \dfrac{1}{4\pi\varepsilon_0}\int \dfrac{\lambda(x)}{r^2}\hat{r}\,dl'$ $\quad$ here, $dl' = dx$, $\lambda(x) = \lambda = \text{const.}$

$E = k\lambda \int_{-L}^{L} \dfrac{-x\hat{x} + z\hat{z}}{(z^2+x^2)^{3/2}}\,dx$ $\quad$ let $u = z^2+x^2 \Rightarrow du = 2x\,dx$

$-\int \dfrac{x\hat{x}}{(z^2+x^2)^{3/2}}\,dx = -\dfrac{\hat{x}}{2}\int_{z^2+L^2}^{z^2+L^2} u^{-3/2}\,du = 0$

$\int_{-L}^{L} \dfrac{dx}{(z^2+x^2)^{3/2}} =$ $\quad$ let $x = z\tan(u) \Rightarrow dx = z\sec^2(u)\,du$



$= \int \dfrac{z\sec^2(u)\,du}{z^3(1+\tan^2(u))^{3/2}} = \dfrac{1}{z^2}\int \dfrac{\sec^2(u)\,du}{\sec^3(u)} = \dfrac{1}{z^2}\int \cos(u)\,du$

$= \dfrac{1}{z^2}\sin(u) = \dfrac{x}{z^2\sqrt{z^2+x^2}}\Big|_{-L}^{L} = \dfrac{2L}{z^2\sqrt{z^2+L^2}}$

So,

$E = k\lambda \dfrac{2L}{z\sqrt{z^2+L^2}}\hat{z}$