

1.56

$$\vec{v} = 6\hat{x} + yz^2\hat{y} + (3y+z)\hat{z}$$

First segment, $x=z=0, y=y \Rightarrow d\vec{l} = dy\hat{y}$

$$\vec{v} \cdot d\vec{l} = yz^2 \text{ by } z=0 \text{ so } \vec{v} \cdot d\vec{l} = 0$$

Second segment,

$$z = -2y + 2 = 2(1-y) \Rightarrow dz = -2dy \quad ; x=0$$

$$d\vec{l} = dy\hat{y} + dz\hat{z} = (y - 2\hat{z})dy$$

$$\vec{v} \cdot d\vec{l} = yz^2 - 2(3y+z) = y(2-2y)^2 - 2(3y+2-2y)$$

$$= y(4 - 8y + 4y^2) - (2y+4) = 4y^3 - 8y^2 + 4y - 2y - 4$$

$$\int_1^0 (4y^3 - 8y^2 + 2y - 4) dy = [y^4 - \frac{8}{3}y^3 + y^2 - 4y]_1^0 = -[1 - \frac{8}{3} + 1 - 4] = 2 + \frac{8}{3}$$

Third segment

$$x=y=0, z=z \Rightarrow d\vec{l} = dz\hat{z}$$

$$\vec{v} \cdot d\vec{l} = (3y+z)dz = z dz$$

$$\int_2^0 z dz = \frac{1}{2}z^2 \Big|_2^0 = 0 - \frac{1}{2}(2)^2 = -2 \text{ so } \oint \vec{v} \cdot d\vec{l} = 2 + \frac{8}{3} - 2 = \frac{8}{3}$$

$$(\vec{\nabla} \times \vec{v}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6 & yz^2 & 3y+z \end{vmatrix} = \hat{i}(3 - 2yz) + \hat{j}(0-0) + \hat{k}(0-0) = (3 - 2yz)\hat{i}$$

$$\int_S (3 - 2yz) dA = \int_0^1 \int_0^{2-2y} (3 - 2yz) dz dy = \int_0^1 [3z - yz^2]_0^{2-2y} dy$$

$$3(2-2y) - y(2-2y)^2 = 6 - 6y - y(4 - 8y + 4y^2) = 6 - 6y - 4y + 8y^2 - 4y^3 = 4y^3 + 8y^2 - 10y + 6$$

$$\Rightarrow \int_0^1 (3 - 2yz) dA = [-\frac{4}{3}y^4 + \frac{8}{3}y^3 - 5y^2 + 6y]_0^1 = -\frac{4}{3} + \frac{8}{3} - 5 + 6 = \frac{8}{3}$$