1.50 (a) $F_1 = x^2\hat{z}$; $F_2 = r\hat{r}$; $F_3 = yz\hat{x} + zx\hat{y} + xy\hat{z}$

$\vec{\nabla}\cdot\vec{F}_1 = 0$

$\vec{\nabla}\cdot\vec{F}_2 = x+y+z$

$$\vec{\nabla}\times F_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & x^2 \end{vmatrix} = -2x\hat{y}$$

$$\vec{\nabla}\times\vec{F}_2 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0$$

So $\vec{F}_1$ is the curl of some vector $\vec{F}_1'$

& $\vec{F}_2$ is the gradient of some scalar f_2

$$f_2 = \left(\tfrac{1}{2}x^2 + g_1(y,z)\right)\hat{x} + \left(\tfrac{1}{2}y^2 + g_2(x,z)\right)\hat{y} + \left(\tfrac{1}{2}z^2 + g_3(x,y)\right)\hat{z}$$

$$\vec{F}_1' = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}$$

$$\vec{F}_1 = \vec{\nabla}\times\vec{F}_1' = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \hat{x}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \hat{y}\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \hat{z}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

$$\frac{\partial F_z}{\partial y} = \frac{\partial F_y}{\partial z}, \quad \frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x}; \quad \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = x^2$$

one solution is

$$\vec{F}_1' = \tfrac{1}{3}x^3\hat{y}$$