1.39

$$\int_V (\vec{\nabla}\cdot\vec{v})\,d\tau = \oint_S \vec{v}\cdot d\vec{a}$$

$$\vec{V}_1 = r^2\hat{r} = v_r\hat{r} + v_\theta\hat{\theta} + v_\phi\hat{\phi} \Rightarrow v_1 = r^2,\ v_\theta = v_\phi = 0$$

$$\vec{\nabla}\cdot\vec{V}_1 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_1) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^4) = \frac{4r^3}{r^2} = 4r$$

$$\int_V (\vec{\nabla}\cdot\vec{V}_1)\,d\tau = \int_V 4r^3 \sin\theta\, dr\,d\phi\,d\theta = \frac{16\pi R^4}{4} = 4\pi R^4$$

$d\vec{a} = d\Omega\, r^2\hat{r} = r^2\sin\theta\, d\theta\, d\phi\,\hat{r}$ ($d\Omega$: solid angle)

$$\vec{V}_1\cdot d\vec{a} = r^4\sin\theta\, d\theta\, d\phi$$

$$\oint_S \vec{v}_1\cdot d\vec{a} = \oint_S R^4\sin\theta\, d\theta\, d\phi = 4\pi R^4$$